Nuclear Reactions: A Challenge for Few- and Many-Body Theory

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Abstract. A current interest in nuclear reactions, specifically with rare isotopes concentrates on their reaction with neutrons, in particular neutron capture. In order to facilitate reactions with neutrons one must use indirect methods using deuterons as beam or target of choice. For adding neutrons, the most common reaction is the (d,p) reaction, in which the deuteron breaks up and the neutron is captured by the nucleus. Those (d,p) reactions may be viewed as a three-body problem in a many-body context. This contribution reports on a feasibility study for describing phenomenological nucleon-nucleus optical potentials in momentum space in a separable form, so that they may be used for Faddeev calculations of (d,p) reactions.

1. Introduction

Today the non-relativistic nuclear three-body system can numerically be solved with high precision in momentum [1] as well as coordinate space [2], including three-nucleon forces. Beyond this, three-body models can play an important role in the description of many-body problems.

Since the full many-nucleon problem presently can only be solved for light nuclei [3, 4, 5, 6], a common approach to treating reactions involving heavier nuclei is concentrate on fewer, relevant degrees of freedom and employ few-body methods. Such an approach is mostly justified for direct reactions, and may be expected to be reasonable if either the projectile fails to significantly penetrate the nucleus (i.e. it is peripheral) or if it penetrates, it moves very fast with respect to the nucleons bound in the nucleus. When a reaction is peripheral, it can be expected that only very few degrees of freedom of the nucleus may be excited due to the relatively low density at the nuclear surface. However, one can argue that in case of attractive optical potentials a reaction will also be mostly peripheral. The projectile has a higher kinetic energy and a shorter wave length inside the nucleus, so that increased oscillations of the wave function inside the nucleus tend do suppress the contribution of the nuclear interior.

2. (d,p) Reactions as Three-Body Problem

Scattering and reaction processes induced by deuterons as projectile are perhaps the most natural three-body problem in the realm of nuclear reactions. The binding energy of the deuteron is so small that its root-mean-square radius (rms) is significantly larger than the range of the nuclear force, meaning that most of the time the neutron and the proton inside the deuteron can be viewed as being outside the range of the interaction. Thus, when a deuteron interacts with a
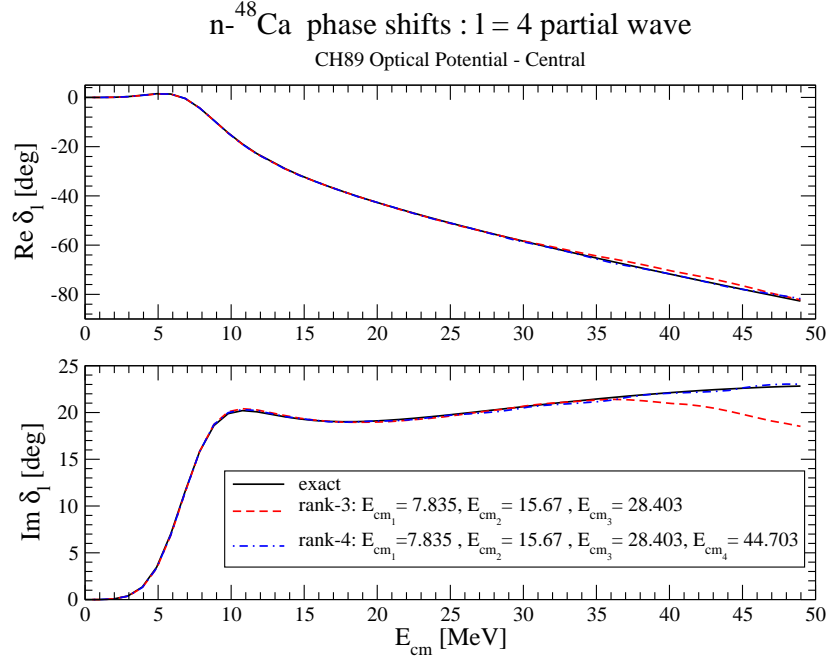
Figure 1. The $l = 4$ phase shift calculated from the central part of the CH89 optical potential [8] for n+^{48}Ca (solid line) together with a rank-3 (dashed line) and rank-4 (dash-dotted line) separable representation as function of the c.m. energy.
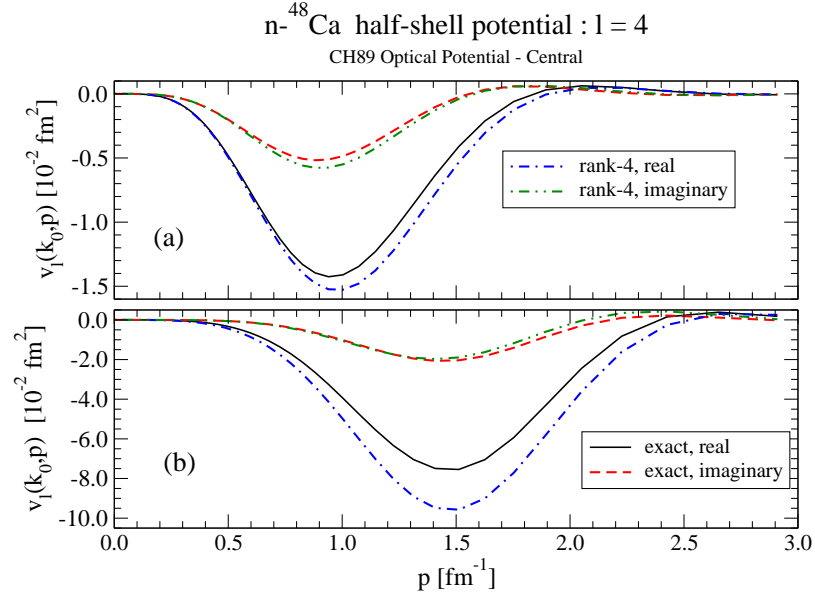
Figure 2. The half-shell potentials $v_{l=4}(k_0, p)$ corresponding to the CH89 central optical potential (exact) together with the rank-4 separable representation at fixed momentum k_0 corresponding to $E_{c.m.} = 4.897$ MeV (a) and $E_{c.m.} = 48.97$ MeV (b).

compact nucleus, one may expect that it will behave like a three-body system consisting of a proton p, a neutron n, and a nucleus A.
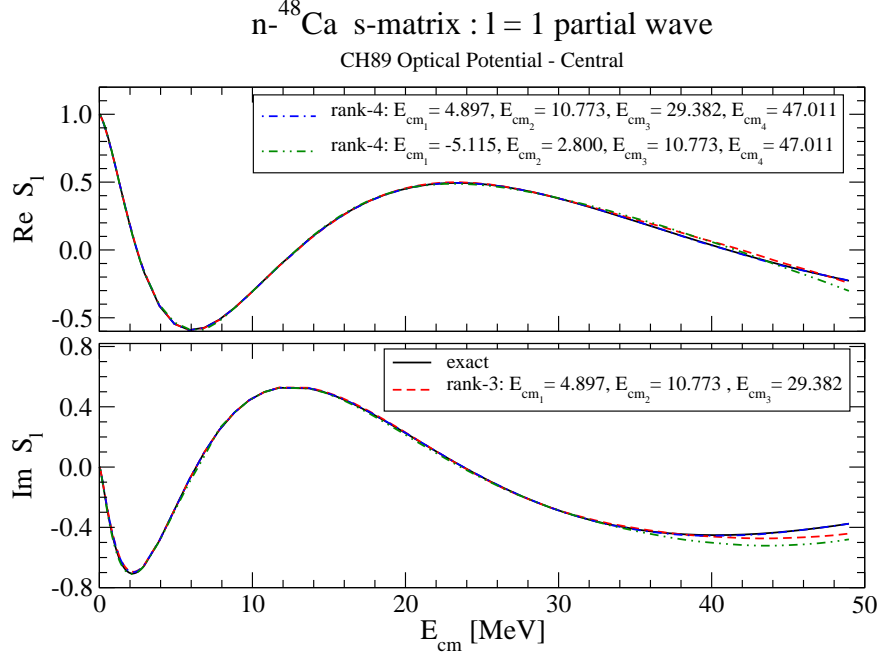
The most obvious three-body effects are rearrangement (stripping followed by pickup) and breakup processes. In order to treat those processes as well as elastic deuteron scattering on the same footing, deuteron-nucleus scattering should be treated as three-body problem making assuming that the system can be described by a three-body Hamiltonian. It should be noted that the extraction of an effective three-body Hamiltonian from the many-body problem is non-trivial. A common approach is using as forces between the neutron and proton (both interacting via the nucleon-nucleon force) neutron and proton-nucleus optical potentials, which in turn are fitted to a large body of elastic scattering data (e.g. [7, 8]). Since these optical potentials are in general energy dependent, a common procedure is to take the nucleon-nucleus optical potentials at half the deuteron incident energy. With this Hamiltonian Faddeev type equations for (d,p) reactions can be set up and solved [9]. Treating nuclear reactions within a Faddeev three-body formulation is at the forefront of developments in nuclear reactions. Existing formulations need to be extended, since the original Faddeev description of a three-body problem does not consider possible internal excitations of the particles active in the scattering process. In a (d,p) reaction with a nucleus, it is definitely possible that the nucleus can be excited during the scattering process. When considering reactions involving exotic nuclei, this may become even more urgent when the exotic nuclei under consideration are loosely bound.

A Faddeev formulation of (d,p) reactions including excitations of the nuclear target has been proposed in Refs. [10, 11]. Solving Faddeev equations is most conveniently carried out in momentum space as coupled integral equations, since those automatically contain the boundary conditions. Thus all interactions in the three-body Hamiltonian need to be given in momentum space. Specifically, a momentum space Faddeev formulation requires as input from the sub-systems, which are here either the deuteron of the neutron-nucleus or proton-nucleus pair, two-body transition matrices (i.e. the interactions summed to all orders) for those sub-systems. It is well known that the numerical effort in solving momentum space Faddeev integral equations is reduced when separable representations of the sub-system transition amplitudes are used. In the case of exited states of the target, separable interactions have the additional important feature that for any resonance as well as for a bound state any two-body transition amplitude is separable. For this reason the formulation in Ref. [10] is based on using separable transition amplitudes for the sub-systems.

3. Separabilization of Nucleon-Nucleus Potentials

Separable representations of nucleon-nucleon (NN) t-matrices are available since the 1980s (see e.g. Refs. [12, 13]). However, as far as neutron or proton-nucleus optical potentials are concerned, there only exist very few separable potentials [11, 14] for light nuclei, which are based on Yamaguchi-type form factors. Those form factors may not be well suited for parameterizing optical potentials for heavier nuclei, for which excellent phenomenological descriptions in terms of Wood-Saxon functions exist (see e.g. [8, 7]). Considering the variety of nuclei for which there is or will be experimental information available from (d,p) reactions, one will need a separabilization procedure that is sufficiently general so that it can be applied to a variety of nucleon-nucleus optical potentials over a wide range of nuclei. In addition, transition amplitudes calculated from phenomenological optical potentials need to be well represented over a wider range of energies.

The separable representation of two-body interactions suggested by Ernst-Shakin-Thaler [15] (EST) looks well suited to achieve this goal. The basic idea of the EST separabilization of a two-body transition amplitude is that one picks a fixed number of energies E_{k_E} for which the transition amplitude is exact, and all other energies are described by interpolation between those points E_{k_E} by the EST separable transition amplitude in which the half-shell amplitudes

n-^{48}Ca s-matrix : $l = 1$ partial wave

CH89 Optical Potential - Central

rank-4: $E_{cm_1} = 4.897$, $E_{cm_2} = 10.773$, $E_{cm_3} = 29.382$, $E_{cm_4} = 47.011$
rank-4: $E_{cm_1} = -5.115$, $E_{cm_2} = 2.800$, $E_{cm_3} = 10.773$, $E_{cm_4} = 47.011$

exact
rank-3: $E_{cm_1} = 4.897$, $E_{cm_2} = 10.773$, $E_{cm_3} = 29.382$

Figure 3. The real (upper panel) and imaginary (lower panel) partial wave s-matrix for $l = 1$ calculated from the central part of the CH89 optical potential [8] for n+^{48}Ca (solid line) as function the c.m. energy. A rank-3 (dashed) and two different rank-4 (dash-dotted and dash-double-dotted) are compared to the exact calculation.

evaluated at E_{k_E} serve as form-factors. The number of energy points E_{k_E} determines the rank of the separable t-matrix. The description of the original t-matrix can in principle be made successively more accurate with increasing support points E_{k_E}.

Though the EST scheme was extensively used to represent NN t-matrices [12, 13], it has to our knowledge not yet been applied to optical potentials for nucleon-nucleus scattering. We report here on a study in which we use the EST scheme to represent the partial wave projected t-matrices for neutron scattering from ^{48}Ca based on the Chapel-Hill CH89 phenomenological global optical potential [8]. For this study we concentrate on the central part of the optical potential and neglect the spin-orbit terms.

According to Ref. [15] a separable potential of arbitrary rank is given as

$$\mathbf{V} = \sum_{i,j} v|\Psi_i\rangle\langle\Psi_i|M|\Psi_j\rangle\langle\Psi_j|v, \tag{1}$$

where $|\Psi_i\rangle$ stands for either a bound-state $|\Psi_B\rangle$ or a scattering state $|\Phi_{k_E}^{(+)}\rangle$. If only bound states are considered, the standard unitary pole approximation (UPA) is recovered. The matrix M is defined and constrained by

$$\delta_{ik} = \sum_j \langle\Psi_i|M|\Psi_j\rangle\langle\Psi_j|v|\Psi_k\rangle = \sum_j \langle\Psi_i|v|\Psi_j\rangle\langle\Psi_j|M|\Psi_k\rangle. \tag{2}$$

In order for the t-matrix elements to have the same structure as the Born term of the Lippmann-Schwinger integral equation, they must be of the form

$$\hat{t}(E) = \sum_{i,j} v|\Psi_i\rangle\tau_{ij}(E)\langle\Psi_j|v . \tag{3}$$
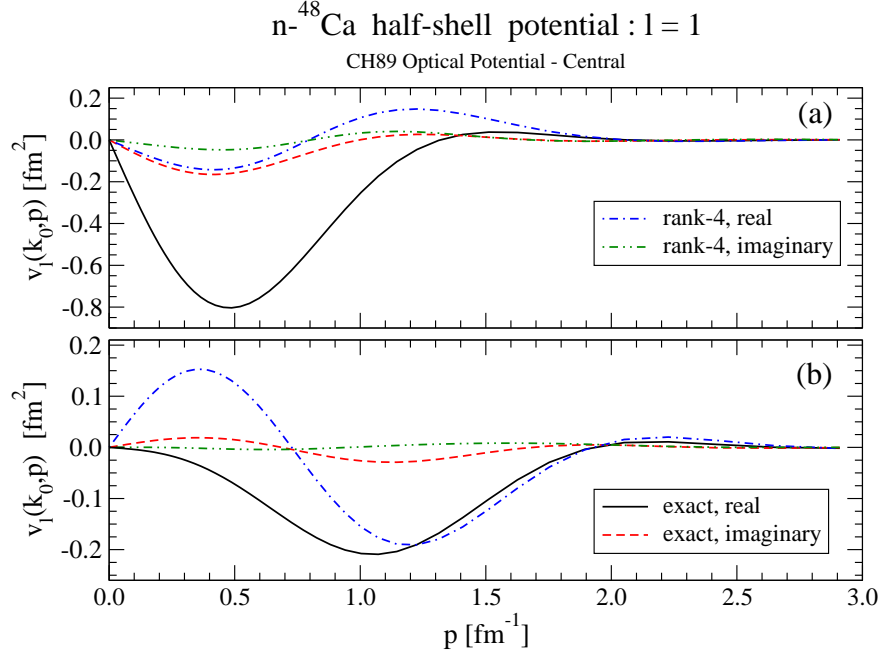
Figure 4. The real (solid) and imaginary (dashed) partial wave projected half-shell CH89 [8] optical potentials for n+^{48}Ca scattering are shown for fixed on-shell momentum k_0 as function of the half-shell momentum p. In (a) the on-shell momentum k_0 corresponds to $E_{c.m.} = 4.897$ MeV, and in (b) to $E_{c.m.} = 48.97$ MeV. A potential leading to a rank-4 separable representation of the corresponding s-matrix is shown by the dashed-dotted line (real part) and dashed-double-dotted line (imaginary part).

It can be shown [15] that the quantities $\tau_{ij}(E)$ fulfill

$$\sum_j \tau_{ij}(E) \langle \Psi_j | v - vg_0(E)v | \Psi_k \rangle = \delta_{ik} \,, \tag{4}$$

and thus can be calculated by inverting the matrix $\langle \Psi_j | v - vg_0(E)v | \Psi_k \rangle$ for given $|\Psi_{j,k}\rangle$, so that finally the momentum space t-matrix elements are given as

$$\hat{t}(p, p', E) = \sum_{ij} \langle p | v_l | \psi_i \rangle \tau_{ij}(E) \langle \psi_j | v_l | p' \rangle \tag{5}$$

The central part of the CH89 optical potential for n+^{48}Ca scattering is attractive and supports either one ($l = 2, 3$) or two ($l = 0, 1$) bound states in the lower partial waves. Partial wave beyond $l = 4$ do not support bound states. For those partial waves the EST support points at with the exactly calculated half-shell t-matrix is used to construct the form factors for the separable expansion are all chosen to be at positive c.m. energies. In Fig. 1 the real and imaginary part of the $l = 4$ phase shift are shown for a rank-3 and a rank-4 approximation and compared to the exactly calculated phase shift. The most prominent structure of the phase shift occurs roughly between 5 and 15 MeV c.m. energy. Thus two support points are needed to fit this structure and one additional at around 30 MeV to describe the flat behavior in that region. If one is interested to describe the phase shift at higher energies, then additional support points can be added without problem.

In Fig. 2 the corresponding half-shell potentials $v_{l=4}(k_0, p)$, calculated using Eq. (1), are shown for two different values of k_0 and compared to the original CH89 optical potential. For the upper panel (a) k_0 is fixed corresponding to $E_{c.m.} = 4.897$ MeV and for the lower panel (b) to $E_{c.m.} = 48.97$ MeV. Since the CH89 optical potential is energy dependent, so are the separable expansions. In the latter, the form factors are energy dependent, which is formally allowed [16]. The original, local Wood-Saxon potential and the non-local rank-4 separable representation are amazingly similar half-shell.

As example for a lower partial wave we consider $l = 1$. This partial wave supports two bound states in its Wood-Saxon well, a ground state at $E_{B_1} = 28.3$ MeV and an excited state at $E_{B_2} = 5.1$ MeV. For a (d,p) reaction only the latter is relevant, since a neutron may be captured into this state. In Fig. 3 the partial wave $l = 1$ s-matrix calculated from the CH89 cental optical potential is shown together with a selection of separable representations. If one is only interested in a representation of the s-matrix for scattering energies, then one can proceed in exactly the same fashion as in the higher partial waves that do not support bound states. A rank-3 and a rank-4 separable representation constructed in this fashion are shown in Fig. 3 as dashed and dash-dotted lines. The rank-4 expansion describes the s-matrix essentially perfectly up to 50 MeV. In a three-body calculation, which takes into account the capture of a neutron into the $l = 1$ bound state close to threshold, the pole structure given by this bound state must be taken into account exactly. In a separable expansion this can be easily accomplished by using the bound state wave function as form factor (as done in UPA approximations) for one of the expansion terms. The dash-double-dotted line in Fig. 3 represents a separable expansion in which one of the terms contains the bound state wave function of the excited state as form factor. Obviously, the positive energy EST support points need to be readjusted. Their values are indicated in Fig. 3. We also tested if the description of the s-matrix depends on introducing the ground state wave function as an additional form factor. However, since its energy is quite far below threshold, the effect on the separable representation for positive energies is negligible.

In Fig. 4 the corresponding partial wave half-shell potentials $v_{l=1}(k_0, p)$ are shown for the central, local CH89 optical potential as function of the half-shell momentum together a non-local, separable rank-4 potential obtained via Eq. (1). As in Fig. 2, an on-shell point k_0 corresponding to $E_{c.m.} = 4.897$ MeV is chosen for panel (a) and to $E_{c.m.} = 48.97$ MeV for panel (b). In this case the both half-shell potentials differ considerably from each other, despite describing the same s-matrix elements. The phenomenon that potentials may show considerable off-shell differences but still lead to very similar t-matrices (and thus s-matrices) has been studied for various NN potentials [17]. Since the information from two-body subsystems which enters Faddeev calculations only enters via t-matrices, differences in potentials do not matter. If the fully off-shell t-matrices show substantial differences in momentum regions important to a Faddeev calculations will need to be investigated further.

4. Summary and Outlook

For describing all aspects of (d,p) reactions on nuclei a three-body Faddeev formulation is currently the most desirable ansatz, since in a Faddeev formulation elastic scattering, breakup and rearrangement processes are treated on the same footing. Considerable progress has been made in solving Faddeev equations in momentum space for (d,p) reactions [9], however there are still major open issues. One of them is the inclusion of target excitations, i.e. the intrinsic excitation of one of the three-body constituents, which is not inherent in the original Faddeev formulation for three-body scattering. A formulation for (d,p) reactions including target excitations has recently been proposed in Ref. [10]. which is based on a separable representation of the transition amplitudes of the two-body sub-systems. While separable representations for the NN sub-system have been developed some time ago, for the neutron- and proton-nucleus sub-system corresponding representations do not exist. In a feasibility study based on neutron

scattering off ^{48}Ca and using only the central piece of a Wood-Saxon based phenomenological optical potential fit [8], we show that for c.m. energies below 50 MeV the s-matrix can be well represented by rank-4 separable transition amplitudes. Future work will include a closer inspection of the fully off-shell behavior of the transition amplitudes derived from local optical potentials compared to their separable representations in momentum regions relevant for solving (d,p) Faddeev equations as well as considering representations for the proton-nucleus optical potential.

References

[1] W. Glöckle, et. al., Phys. Rep. **274**, 107 (1996).
[2] P. Barletta and A. Kievsky, Few Body Syst. **45**, 25 (2009).
[3] S. C. Pieper, K. Varga and R. B. Wiringa, Phys. Rev. C **66**, 044310 (2002).
[4] S. C. Pieper, Riv. Nuovo Cim. **31**, 709 (2008), [arXiv:0711.1500 [nucl-th]].
[5] P. Navratil and S. Quaglioni, Phys. Rev. C **83**, 044609 (2011).
[6] P. Navratil, R. Roth and S. Quaglioni, Phys. Rev. C **82**, 034609 (2010).
[7] S. P. Weppner, R. B. Penney, G. W. Diffendale and G. Vittorini, Phys. Rev. C **80**, 034608 (2009).
[8] R. L. Varner, W. J. Thompson, T. L. McAbee, E. J. Ludwig and T. B. Clegg, Phys. Rept. **201**, 57 (1991).
[9] A. Deltuva and A. C. Fonseca, Phys. Rev. C **79**, 014606 (2009).
[10] A. M. Mukhamedzhanov, V. Eremenko and A. I. Sattarov, arXiv:1206.3791 [nucl-th].
[11] E. O. Alt, L. D. Blokhintsev, A. M. Mukhamedzhanov and A. I. Sattarov, Phys. Rev. C **75**, 054003 (2007).
[12] J. Haidenbauer and W. Plessas, Phys. Rev. C **30**, 1822 (1984).
[13] J. Haidenbauer, Y. Koike and W. Plessas, Phys. Rev. C **33**, 439 (1986).
[14] K. Miyagawa and Y. Koike, Prog. Theor. Phys.**82**, 329, (1989).
[15] D. J. Ernst, C. M. Shakin and R. M. Thaler, Phys. Rev. C **8**, 46 (1973).
[16] D. J. Ernst, J. T. Londergan, E. J. Moniz and R. M. Thaler, Phys. Rev. C **10**, 1708 (1974).
[17] E. F. Redish and K. Stricker-Bauer, Phys. Rev. C **36**, 513 (1987).